Homework #1 (10 points) - Show all work on the following problems:

Problem 1 (1 point): Find the gradients of the following functions:
(a) $f(x, y, z) = x^2 + y^3 + z^4$.
(b) $f(x, y, z) = e^x \sin(y) \ln(z)$

Problem 2 (1 point): Find the divergence and the curl of the following function:

$$\vec{A}(x,y,z) = xy\hat{x} + 2yz\hat{y} + 3zx\,\hat{z}$$

Problem 3 (1 point): Prove that the divergence of a curl is always zero.

Problem 4 (3 points): Calculate the line integral of the function $\vec{A}(x,y,z) = x^2\hat{x} + 2yz\hat{y} + y^2\hat{z}$ over each of the following three paths:

(a) (0,0,0) -> (1,0,0) -> (1,1,0) -> (1,1,1)
(b) (0,0,0 -> (0,0,1) -> (0,1,1) -> (1,1,1)
(c) Along the direct straight line from (0,0,0) to (1,1,1)

Problem 5 (2 points): Check Stokes' theorem $\iint \left(\nabla\times\vec{A}\right)\cdot\overrightarrow{da} = \oint \vec{A}\cdot\overrightarrow{dl}$ for the function $\vec{A}(x,y,z) = xy\hat{x} + 2yz\hat{y} + 3zx\hat{z}$, for a triangular area with ordered vertices (0,0,0), (0,2,0), and (0,0,2).

Problem 6 (2 points): Check the divergence theorem $\iiint \left(\nabla\cdot\vec{A}\right) dV = \oiint \vec{A}\cdot\overrightarrow{da}$ for the function $\vec{A}(r,\theta,\phi) = r^2\hat{r}$, using as your volume a sphere of radius R centered at the origin.